Essentials Algebra 2 Paper Packet

Weeks 1 & 2:

  Standard 55: Multiply and simplify radical expressions.

  Standard 56: Divide and simplify radicals expressions.

Students will copy the notes from the key to their own copy.

Notes: Simplifying Radicals Key: Simplifying Radicals

Then they will need to work on the homework problems.

Homework: Simplifying Radicals

Students will copy the notes from the key to their own copy.

Notes: Multiplying and Dividing Radicals Key: Multiplying and Dividing Radicals

Then they will need to work on the homework problems.

Homework: Multiplying and Dividing Radicals

Weeks 3 - 5:

  Standard 57: Add and subtract radicals.

  Standard 58: Multiply binomial radical expressions.

  Standard 59: Rationalize the denominator by multiplying by the conjugate.

Students will copy the notes from the key to their own copy.

Notes: Add, Subtract, Multiply, and Rationalize Radicals Key: Add, Subtract, Multiply, and Rationalize

Then they will need to work on the homework problems.

  Homework: Add, Subtract, Multiply and Rationalize Radicals

  More Practice: Operations with Radicals

Week 6:

  Formative Assessment: Check what you know! Students should work on these problems and see where their current understanding is at. Work on this review and see what standards need the most help!

Review Standards (55-59)

$$\sqrt[n]{a}$$

Index-

Radical-

Radicand-

For each radical, find the index and the radicand.

Ex. $\sqrt[4]{128n^8}$ Ex. $\sqrt{512x^2}$ Ex. $\sqrt[3]{16xy}$

Steps for Simplifying Radicals

1.   Write the prime factorization of your radicand. (Divide by prime numbers ex. 2, 3, 5, 7, etc)

2.   Determine the index of the radical.

3.   If the index is 2, circle identical numbers or variables. If the index is 3, circle identical numbers or variables.

4.   The number or variable from the circled group will show up outside the radical symbol one time.

5.   Anything left uncircled will stay under the radical. If everything is circled, then the radical will disappear.

6.   Multiply the numbers and variables outside the radical together.

Simplifying Radical Expressions

Ex. $\sqrt{8x^7}$

Ex. $\sqrt[3]{27c^6}$

Ex. $\sqrt[4]{32x^4y^4}$

Ex. $\sqrt[3]{108a^4b^6}$

Ex. $\sqrt[4]{64x^6y^2}$

Ex. $\sqrt{50x^5y^4}$

$$\sqrt[n]{a}$$

Index- the degree of the root

Radical- the symbol for the root

Radicand- # or variable under the radical

For each radical, find the index and the radicand.

Ex. $\sqrt[4]{128n^8}$ Ex. $\sqrt{512x^2}$ Ex. $\sqrt[3]{16xy}$

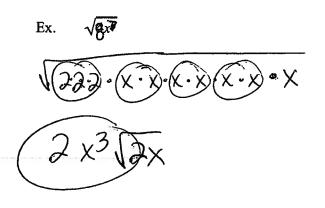Index = 4   Index = 2   Index = 3

radicand : $128n^8$ radicand: $512x^2$ radicand = $16xy$

Steps for Simplifying Radicals

1.   Write the prime factorization of your radicand. (Divide by prime numbers ex. 2, 3, 5, 7, etc)

2.   Determine the index of the radical.

3.   If the index is 2, circle identical numbers or variables. If the index is 3, circle identical numbers or variables.

4.   The number or variable from the circled group will show up outside the radical symbol one time.

5.   Anything left uncircled will stay under the radical. If everything is circled, then the radical will disappear.

6.   Multiply the numbers and variables outside the radical together.

Simplifying Radical Expressions

Ex. $\sqrt{8x^7}$

$\sqrt{(2\cdot2\cdot2)\cdot(x\cdot x)\cdot(x\cdot x)\cdot(x\cdot x)\cdot x}$

$2x^3\sqrt{2x}$

Ex. $\sqrt[3]{27c^6}$

$3 \quad 9$

$3 \quad 3$

$\sqrt[3]{(3\cdot3\cdot3)\cdot(c\cdot c\cdot c)\cdot(c\cdot c\cdot c)}$

$3c^2$

Ex. $\sqrt[4]{32x^4y^4}$

$4 \quad 8$

$2 \quad 2 \quad 2 \quad 4$

$2 \quad 2$

$\sqrt[4]{2\cdot2\cdot2\cdot2\cdot2\cdot(x\cdot x\cdot x\cdot x)\cdot(y\cdot y\cdot y\cdot y)}$

$2xy\sqrt[4]{2}$

Ex. $\sqrt[3]{108a^3b^6}$

$2 \quad 54$

$9 \quad 6$

$3\,3 \quad 2\,3$

$\sqrt[3]{2\cdot2\cdot(3\cdot3\cdot3)\cdot(a^3)\cdot a\cdot(b^3)\cdot(b^3)}$

$3ab^2\sqrt[3]{4a}$

Ex. $\sqrt[4]{64x^6y^2}$

$8 \quad 8$

$4 \quad 2 \quad 2 \quad 4$

$2 \quad 2$

$2 \quad 2$

$\sqrt[4]{2\cdot2\cdot(2\cdot2\cdot2\cdot2)\cdot(x^4)\cdot x^2\cdot y^2}$

$2x\sqrt[4]{4x^2y^2}$

Ex. $\sqrt{50x^5y^4}$

$10 \quad 5$

$5 \quad 2$

$\sqrt{5\cdot5\cdot2\cdot(x^2\cdot x^2)\cdot x\cdot(y^2\cdot y^2)}$
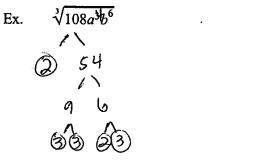
$5x^2y^2\sqrt{2x}$

Find the index and the radicand.

1. $\sqrt{16x^2}$

2. $\sqrt[5]{32y^{10}}$

3. $\sqrt[3]{81x^9}$

Simplify each radical.

4. $\sqrt{24}$

5. $\sqrt{98k^5}$

6. $\sqrt[3]{48m^3}$

7. $\sqrt[4]{128x^7y^7}$

8. $\sqrt[3]{56x^5y}$

9. $\sqrt[3]{162x^4y^2}$

10. What simplifies into $2mn^2\sqrt[3]{5mn^2}$?

Multiplying Radical Expressions

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

Does it work?   Ex. $\sqrt{16} \cdot \sqrt{9}$

Multiplying Radicals

Ex. $\sqrt{2} \cdot \sqrt{8}$ Ex. $\sqrt[3]{-5} \cdot \sqrt[3]{25}$ Ex. $\sqrt{-2} \cdot \sqrt{8}$ Ex. $\sqrt{3} \cdot \sqrt[3]{4}$

Multiplying Radical Expressions

Ex. $\sqrt[3]{54x^2y^3} \cdot \sqrt[3]{5x^3y^4}$ Ex. $\sqrt{5xy} \cdot \sqrt{40x^3y}$

Multiplying Radical Expressions

Ex. $3\sqrt{7x^3} \cdot 2\sqrt{21x^3y^2}$

Ex. $-4\sqrt[3]{16n^5} \cdot 5\sqrt[3]{2n^2}$

Dividing Radical Expressions

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

Ex. $\dfrac{\sqrt[3]{162x^5}}{\sqrt[3]{3x^2}}$

Ex. $\dfrac{\sqrt{12x^4}}{\sqrt{3x}}$

Rationalizing the Denominator

Ex. $\dfrac{\sqrt{x^3}}{\sqrt{5xy}}$

Ex. $\sqrt[3]{\dfrac{2}{3x}}$

Multiplying Radical Expressions

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

* In order to simplify, they need to have the same <u>index</u>!

Does it work?   Ex. $\sqrt{16} \cdot \sqrt{9}$ or $\sqrt{16 \cdot 9}$

$4 \cdot 3$ $\sqrt{144}$

12 12

Multiplying Radicals

Ex. $\sqrt{2} \cdot \sqrt{8}$ Ex. $\sqrt[3]{-5} \cdot \sqrt[3]{25}$ Ex. $\sqrt{-2} \cdot \sqrt{8}$ Ex. $\sqrt{3} \cdot \sqrt[3]{4}$

$\sqrt{16}$ $\sqrt[3]{-125}$ $\sqrt{-16}$ Not possible

4 -5 Not possible   Don't have the same index.

Multiplying Radical Expressions

Ex. $\sqrt[3]{54x^2y^3} \cdot \sqrt[3]{5x^3y^4}$ Ex. $\sqrt{5xy} \cdot \sqrt{40x^3y}$

$\sqrt[3]{270x^5y^7}$ $\sqrt{200x^4y^2}$

$\sqrt[3]{2(3 \cdot 3 \cdot 3)5(x^3)x^2(y^3)(y^3)y}$ $\sqrt{2 \cdot 2(5 \cdot 5) \cdot 2(x^2)(x^2)y^2}$

$3\overline{\smash{)}3}$ $\boxed{3xy^2\sqrt[3]{10x^2y}}$ $2\overline{\smash{)}2}$ $2 \cdot 5 \cdot x \cdot x \cdot y\sqrt{2}$

$3\overline{\smash{)}9}$ $2\overline{\smash{)}4}$ $\boxed{10x^2y\sqrt{2}}$

$3\overline{\smash{)}27}$ $5\overline{\smash{)}20}$

$2\overline{\smash{)}54}$ $5\overline{\smash{)}100}$

$5\overline{\smash{)}270}$ $2\overline{\smash{)}200}$

Multiplying Radical Expressions

Ex. $\quad 3\sqrt{7x^3} \cdot 2\sqrt{21x^3y^2}$

$$6\sqrt{147x^6y^2}$$

$$6\sqrt{3 \cdot \boxed{7 \cdot 7} \cdot \boxed{x^2}\boxed{x^2}\boxed{x^2}\cdot\boxed{y^2}}$$

$$6 \cdot 7 \cdot x \cdot x \cdot x \cdot y \sqrt{3}$$

$$\boxed{42x^3 y \sqrt{3}}$$

$$7\,\overline{)\,7}^{\,1}$$
$$7\,\overline{)\,49}$$
$$3\,\overline{)\,147}$$

Ex. $\quad -4\sqrt[3]{16n^5} \cdot 5\sqrt[3]{2n^2}$

$$-20\sqrt[3]{32n^7}$$

$$-20\sqrt[3]{\boxed{2\cdot2\cdot2}\cdot2\cdot2 \cdot \boxed{n^3}\boxed{n^3}\cdot n}$$

$$-20 \cdot 2 \cdot n \cdot n \sqrt[3]{4n}$$

$$-40n^2\sqrt[3]{4n}$$

$$2\,\overline{)\,2}^{\,1}$$
$$2\,\overline{)\,4}$$
$$2\,\overline{)\,8}$$
$$2\,\overline{)\,16}$$
$$2\,\overline{)\,32}$$

Dividing Radical Expressions

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b\neq0$, then $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

ex. $\quad \dfrac{\sqrt{36}}{\sqrt{25}} = \dfrac{6}{5} \qquad \sqrt{\dfrac{36}{25}} = \dfrac{6}{5}$

Ex. $\quad \dfrac{\sqrt[3]{162x^5}}{\sqrt[3]{3x^2}}$

$$\sqrt[3]{54x^3}$$

$$\sqrt[3]{2\cdot\boxed{3\cdot3\cdot3}\cdot\boxed{x^3}}$$

$$3x\sqrt[3]{2}$$

$$3\,\overline{)\,3}^{\,1}$$
$$3\,\overline{)\,9}$$
$$3\,\overline{)\,27}$$
$$2\,\overline{)\,54}$$

Ex. $\quad \dfrac{\sqrt{12x^4}}{\sqrt{3x}}$

$$\sqrt{\dfrac{12x^4}{3x}}$$

$$\sqrt{4x^3}$$

$$\sqrt{\boxed{2\cdot2}\cdot\boxed{x^2}\cdot x}$$

$$2x\sqrt{x}$$

$$2\,\overline{)\,2}^{\,1}$$
$$2\,\overline{)\,4}$$

Rationalizing the Denominator

Ex. $\dfrac{\sqrt{x^3}}{\sqrt{5xy}}$

Ex. $\sqrt[3]{\dfrac{2}{3x}}$

$\sqrt{\dfrac{x^3}{5xy}}$

$\dfrac{\sqrt[3]{2}}{\sqrt[3]{3x}} \cdot \dfrac{\sqrt[3]{3x}}{\sqrt[3]{3x}} \cdot \dfrac{\sqrt[3]{3x}}{\sqrt[3]{3x}}$

$\sqrt{\dfrac{x^2 \cdot}{5y}}$

$\dfrac{\sqrt[3]{2 \cdot 3x \cdot 3x}}{\sqrt[3]{3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x}}$

$\dfrac{\sqrt{x^2}}{\sqrt{5y}} \cdot \dfrac{\sqrt{5y}}{\sqrt{5y}}$

$\dfrac{\sqrt{5x^2y}}{\sqrt{25y^2}}$

$\boxed{\dfrac{\sqrt[3]{18x^2}}{3x}}$

$\dfrac{\sqrt{5x^2y}}{\sqrt{5 \cdot 5 \cdot y^2}}$

$\boxed{\dfrac{x\sqrt{5y}}{5y}}$

Essential Algebra 2   Multiply and Divide Radicals HW   Name:

Multiply if possible. Then, simplify.

1. $\sqrt{8} \cdot \sqrt{32}$

2. $\sqrt[3]{5} \cdot \sqrt{5}$

3. $\sqrt[3]{9} \cdot \sqrt[3]{-81}$

4. $\sqrt{7x^5} \cdot \sqrt{42xy^9}$

5. $3\sqrt[3]{5y^3} \cdot 2\sqrt[3]{50y^4}$

6. $4\sqrt{2x} \cdot 5\sqrt{6xy^2}$

7. $-\sqrt[3]{2x^2y^2} \cdot 2\sqrt[3]{15x^5y}$

Divide and simplify.

8. $\dfrac{\sqrt{48x^3}}{\sqrt{3xy^2}}$

9. $\dfrac{\sqrt{56x^5y^5}}{\sqrt{7xy}}$

10. $\dfrac{\sqrt[3]{250x^7y^3}}{\sqrt[3]{2x^2y}}$

11. $\dfrac{\sqrt{5}}{\sqrt{8x}}$

12. $\dfrac{\sqrt[3]{x}}{\sqrt[3]{2}}$

13. $\dfrac{\sqrt{5x^4y}}{\sqrt{2x^2y^3}}$

Essential Algebra 2   Add, Subtract, and Multiply Radical Expressions Notes   Name:

Like Radicals-

Adding and Subtracting Radical Expressions

Ex. $5\sqrt[3]{x} - 2\sqrt[3]{x}$

Ex. $2\sqrt{7} + 3\sqrt{7}$

Ex. $7\sqrt[4]{5} - 2\sqrt[4]{3}$

Ex. $4\sqrt{xy} + 5\sqrt{xy}$

Simplifying Before Adding or Subtracting

Ex. $6\sqrt{18} + 4\sqrt{8} - 3\sqrt{72}$

Ex. $\sqrt{48} + 4\sqrt{12} - 5\sqrt{27}$

Multiplying Binomial Radical Expressions

Ex. $(3 + 2\sqrt{5})(2 + 4\sqrt{5})$

Ex. $(\sqrt{2} - \sqrt{3})^2$

Multiplying Conjugates

Ex. $(2 + \sqrt{3})(2 - \sqrt{3})$

Ex. $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$

Rationalizing Binomial Radical Denominators

Ex. $\dfrac{3 + \sqrt{5}}{1 - \sqrt{5}}$

Ex. $\dfrac{6 + \sqrt{15}}{4 - \sqrt{15}}$

Like Radicals- are radical expressions that have the same index and the same radicand.

ex. $\sqrt[3]{7}$ and $5\sqrt[3]{7}$

Adding and Subtracting Radical Expressions

Ex. $5\sqrt[3]{x} - 2\sqrt[3]{x}$

$(5-2)\sqrt[3]{x}$

$3\sqrt[3]{x}$

Ex. $2\sqrt{7} + 3\sqrt{7}$

$(2+3)\sqrt{7}$

$5\sqrt{7}$

Ex. $7\sqrt[4]{5} - 2\sqrt[4]{3}$

can't combine,
they have different
radicands.

Ex. $4\sqrt{xy} + 5\sqrt{xy}$

$(4+5)\sqrt{xy}$

$9\sqrt{xy}$

Simplifying Before Adding or Subtracting

Ex. $6\sqrt{18} + 4\sqrt{8} - 3\sqrt{72}$

Index = 2

$\sqrt{18} = \sqrt{2 \cdot (3 \cdot 3)} = 3\sqrt{2}$

$\sqrt{8} = \sqrt{(2 \cdot 2) \cdot 2} = 2\sqrt{2}$

$\sqrt{72} = \sqrt{(3 \cdot 3)(2 \cdot 2) \cdot 2} = 6\sqrt{2}$

$6 \cdot 3\sqrt{2} + 4 \cdot 2\sqrt{2} - 3 \cdot 6\sqrt{2} = (18 + 8 - 18)\sqrt{2} = 8\sqrt{2}$

Ex. $\sqrt{48} + 4\sqrt{12} - 5\sqrt{27}$

Index = 2

$\sqrt{48} = \sqrt{(2 \cdot 2)(2 \cdot 2) \cdot 3} = 4\sqrt{3}$

$4\sqrt{12} = 4\sqrt{(2 \cdot 2) \cdot 3} = 8\sqrt{3}$

$5\sqrt{27} = 5\sqrt{(3 \cdot 3) \cdot 3} = 15\sqrt{3}$

$4\sqrt{3} + 8\sqrt{3} - 15\sqrt{3} = -3\sqrt{3}$

Multiplying Binomial Radical Expressions

Ex. $(3 + 2\sqrt{5})(2 + 4\sqrt{5})$

F: $3 \cdot 2 = 6$

O: $3 \cdot 4\sqrt{5} = 12\sqrt{5}$

I: $2\sqrt{5} \cdot 2 = 4\sqrt{5}$

L: $2\sqrt{5} \cdot 4\sqrt{5} = 40$

$12\sqrt{5} + 4\sqrt{5} + 6 + 40$

$\boxed{16\sqrt{5} + 46}$

Multiplying Conjugates

Ex. $(2 + \sqrt{3})(2 - \sqrt{3})$

$4 - 2\sqrt{3} + 2\sqrt{3} - 3$

$4 - 3 = \boxed{1}$

Ex. $(\sqrt{2} - \sqrt{3})^2$

$(\sqrt{2} - \sqrt{3})(\sqrt{2} - \sqrt{3})$

F: $\sqrt{2} \cdot \sqrt{2} = 2$

O: $\sqrt{2} \cdot \sqrt{3} = -\sqrt{6}$

I: $-\sqrt{3} \cdot \sqrt{2} = -\sqrt{6}$

L: $-\sqrt{3} \cdot \sqrt{3} = 3$

$2 + 3 - \sqrt{6} - \sqrt{6}$

$\boxed{5 - 2\sqrt{6}}$

Ex. $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$

$5 + \sqrt{10} - \sqrt{10} - 2$

$5 - 2 = \boxed{3}$

Rationalizing Binomial Radical Denominators

Ex. $\dfrac{3 + \sqrt{5}}{1 - \sqrt{5}} \cdot \dfrac{1 + \sqrt{5}}{1 + \sqrt{5}}$

$\dfrac{3 + 3\sqrt{5} + \sqrt{5} + 5}{1 - 5} =$

$\dfrac{8 + 4\sqrt{5}}{-4} : \dfrac{8}{-4} + \dfrac{4\sqrt{5}}{-4}$

$= \boxed{-2 - \sqrt{5}}$

Ex. $\dfrac{6 + \sqrt{15}}{4 - \sqrt{15}} \cdot \dfrac{4 + \sqrt{15}}{4 + \sqrt{15}}$

$\dfrac{24 + 6\sqrt{15} + 4\sqrt{15} + 15}{16 - 15}$

$\dfrac{39 + 10\sqrt{15}}{1}$

$\boxed{39 + 10\sqrt{15}}$

Essential Algebra 2   Add, Subtract, and Multiply Radical Expressions HW   Name:

Add or subtract if possible.

1. $5\sqrt{6} + \sqrt{6}$

2. $4\sqrt[3]{3} + 4\sqrt{3}$

3. $3\sqrt{x} - 5\sqrt{x}$

Simplify.

4. $\sqrt{24} + \sqrt{96}$

5. $6\sqrt{18} + 3\sqrt{50}$

6. $14\sqrt{20} - 3\sqrt{125}$

7. $3\sqrt[3]{81} - 2\sqrt[3]{54}$

Multiply.

8. $(3 + \sqrt{5})(1 + \sqrt{5})$

9. $(3 - 4\sqrt{2})(5 - 6\sqrt{2})$

10. $(\sqrt{3} + \sqrt{5})(\sqrt{3} + \sqrt{5})$

11. $(5 - \sqrt{11})(5 + \sqrt{11})$

Rationalize each denominator. Simplify the answer.

12. $\dfrac{4}{1 + \sqrt{3}}$

13. $\dfrac{5 + \sqrt{3}}{2 - \sqrt{3}}$

Essential Algebra 2   Add, Subtract, and Multiply Radicals (Part 2)   Name:

Simplify.

1. $3\sqrt{18} - 2\sqrt{2}$

2. $2\sqrt{20} - \sqrt{20} + 3\sqrt{20} - 2\sqrt{45}$

3. $-2\sqrt{3} + 3\sqrt{27}$

4. $-3\sqrt{2} + 3\sqrt{20} - 3\sqrt{8}$

Multiply.

5. $(5 + \sqrt{13})(7 + \sqrt{13})$

6. $(8 - 2\sqrt{3})(1 - 10\sqrt{3})$

Divide

7. $\dfrac{\sqrt[3]{128x^7y^3}}{\sqrt[3]{8x^3}}$

8. $\dfrac{\sqrt[4]{567x^{10}y^8}}{\sqrt[4]{7xy^3}}$

Rationalize each denominator. Simplify the answer.

9. $\dfrac{11}{8-\sqrt{2}}$

10. $\dfrac{2+\sqrt{19}}{-8+\sqrt{19}}$

Standard 55: **Multiply if possible. Then, simplify.**

1. $\sqrt{35x^3} \cdot \sqrt{7x^2}$

2. $\sqrt[3]{16x^7y^2} \cdot \sqrt[3]{8xy^5}$

Standard 56: **Divide and simplify.**

3. $\dfrac{\sqrt{100x^{10}y^6}}{\sqrt{2x^5y^2}}$

4. $\dfrac{\sqrt[3]{160x^3y^7}}{\sqrt[3]{2x^2y}}$

Standard 57: **Add or subtract each radical expression.**

5. $15\sqrt[4]{2xy} - 10\sqrt[4]{2xy}$

6. $\sqrt{18} + \sqrt{162} + \sqrt{32}$

Standard 58

Multiply.

7. $(\sqrt{10} - 6)(\sqrt{10} + 9)$

8. $(\sqrt{3} - \sqrt{8})(\sqrt{3} - \sqrt{8})$

Standard 59

Rationalize each denominator. Simplify the answer.

9. $\dfrac{7}{3+\sqrt{11}}$

10. $\dfrac{2+\sqrt{5}}{8-\sqrt{5}}$